Summary

In this meeting, I was trying to explain the different components of the Hamilton Jacobian Belman Equation (HJB) with a specific example of an Inverted Pendulum. From my point of view, HJB includes terminal conditions, running costs, and system dynamics. The most important term that was discussed in the meeting was Terminal cost in “A General Safety Framework for Learning-Based Control in Uncertain Robotic Systems” and “Data-Driven Safety Filters” assigned as the signed distance function. I am confused by how we choose the boundary for this sign function and the point to measure Euclidean as the signed distance function needs these two independent variables to define. Dr. Petrik also mentioned that this singed distance is the approximation of the step function that is zero or one to determine the safe set. From my point of view, as this function has negative and positive values and since we are looking for a safe set, it should have more attention on the super-level set of this function. Added to this, I failed to find out how HJB can be solved with this terminal condition since the problem is to calculate this boundary that distinguishes whether we are in the safe set or not. In my simulations, I considered the terminal condition as just the Euclidian norm from the origin to the boundary of the feasible set.

Added to this, I also showed the Hamilton Jacobian Isaac (HJI) equation, which did not consider the running cost in this equation, and directly the terminal condition is subtracted from the V(x, t). Also, since I mentioned this equation from different references, the notation caused a lot of confusion and Dr. Begum suggested that I not cut an equation from different references to cause more confusion.

Dr. Petrik also asked why we are choosing this signed distance function, and I have no answer for it for now he also mentioned to solve the HJB, we are solving the running cost in a simple case that includes the terminal cost and stage cost.

Dr. Yoon pointed out that he could not follow what I did and what these figure's integration of value functions also, he mentioned that I should more focus on having a clear mapping between the theory’s components to the example to avoid confusion. Moreover, as the base of all these concepts is dynamic programming (DP), I should establish a solid background in DP.

Dr. Begum had another suggestion to change the system to a simplified quadrotor model, which is discussed in “A General Safety Framework for Learning-Based Control in Uncertain Robotic Systems”.

I should note that Dr. Petrik had another suggestion that I forgot in detail, but I would like to ask him to remind me of his suggestion.

Finally, in the last part of the meeting, Dr. Begum and Dr. Yoon suggested focusing more on theory and having a tutorial on reachability set for the next meeting. Also, she mentioned Nagumo's theorem as a fundamental block.

I argued that safety in this context means satisfying the state and input constraints and Model predictive control can satisfy this and Data-Driven Safety Filters, even if the ideal case is MPC. However, Dr. Yoon mentioned that the MPC cannot guarantee the solution.

I also mentioned that solving HJB or HJI seems too much effort to find the safe set and controller barrier despite being so conservative, but it is straightforward. Dr. Begum mentioned that finding the H function is not easy for different cases.

Dr. Begum asked me about my idea, and I am in the learning process, I said after every meeting, my mind is changing about my idea but what I know that is the initial condition is so important in all approaches, and I like to see the safety problem as local safety area around the initial condition and defined a safety notation inside the running cost.

From the last meeting, I think still what Hamilton equation is unclear to me in this context and I would like to know more about it, I also think it’s time to select one of these frameworks for my Ph.D. thesis or look at safety from a risk perspective. I like to ask some questions and ask their opinion on which framework and why and then make my decision.